Name..ID...........................Section......1........Seat No............

Sirindhorn International Institute of Technology
Thammasat University

Midterm Examination: Semester 2 / 2016

Course Title: ECS452 Digital Communication Systems

Instructor: Asst. Prof. Dr.Prapun Suksompong

Date/Time: March 24, 2017 / 9:00 - 11:00

Instructions:

- This examination has.....6....pages (including this cover page).
- Conditions of Examination:
  ....Closed book
  (No dictionary, ☐ No calculator ☑ Calculator (e.g. FX-991) allowed)
  ....✓..**Semi-Closed book** (.......1........sheet(s) ☑ 1 page ☐ both sides of A4 paper note)
  This sheet must be hand-written.
  Write your name and ID in the top right corner of the page (in portrait orientation).
  Do not modify (,e.g., add/underline/highlight) content on the sheet inside the exam room.
  It should be **submitted with the exam**.
  Other requirements are specified on the course web site. (-10 pt for not following the requirements.)
- **Read these instructions and the questions carefully**.
- Students are not allowed to be out of the examination room during examination. Going to the restroom may result in score deduction.
- Turn off all communication devices and place them with other personal belongings in the area designated by the proctors or outside the test room.
- Write your name, student ID, and seat number clearly in the spaces provided on the top of this sheet. Then, write your **first name and the last three digits of your ID** in the spaces provided on the top of each page of your examination paper, starting from page 2.
- The back of each page will not be graded; it can be used for calculations of problems that do not require explanation.
- The examination paper is not allowed to be taken out of the examination room. Also, do not remove the staple. Violation may result in score deduction.
- Unless instructed otherwise**, *write down all the steps*** that you have done to obtain your answers.
  - When applying formula(s), state clearly which formula(s) you are applying before plugging-in numerical values.
  - You may not get any credit even when your final answer is correct without showing how you get your answer.
  - Formula(s) not discussed in class can be used. However, derivation must also be provided.
  - **Exceptions**:
    - Problems that are labeled with “ENRPr” (Explanation is not required for this problem.)
    - Parts that are labeled with “ENRPa” (Explanation is not required for this part.)

    These problems/parts are graded solely on your answers. There is no partial credit and it is not necessary to write down your explanation. Usually, spaces (boxes or cells in a table or rows of dashes) will be provided for your answers. “WACSP” stands for “write your answer(s) in the corresponding space(s) provided”.
- When not explicitly stated/defined, all notations and definitions follow ones given in lecture.
- Some points are reserved for *accuracy* of the answers and also for reducing answers into their *simplest* forms. Watch out for roundoff error. The error in your final answer should not exceed 0.1%.
- Points marked with * indicate challenging problems.
- Do not cheat. Do not panic. **Allocate your time wisely.**
- Don’t forget to submit your fist online self-evaluation form by the end of today.

Problem 1. (12 pt) [ENRPr] Consider two codes (for source coding) below. The left column is for Code A. The right column is for Code B. The first row defines these codes via their codebooks.

<table>
<tr><td>Codebook for Code A:
<table><tr><td>x</td><td>a</td><td>e</td><td>c</td><td>n</td><td>t</td></tr><tr><td>$c(x)$</td><td>10</td><td>01</td><td>11</td><td>000</td><td>100</td></tr></table></td><td>Codebook for Code B:
<table><tr><td>x</td><td>a</td><td>e</td><td>c</td><td>n</td><td>t</td></tr><tr><td>$c(x)$</td><td>1</td><td>00</td><td>010</td><td>0111</td><td>01101</td></tr></table></td></tr>
<tr><td>The <u>source alphabet</u> for Code A is</td><td>The <u>code alphabet</u> for Code B is</td></tr>
<tr><td>Is Code A nonsingular?</td><td>Is Code B nonsingular?</td></tr>
<tr><td>Is Code A prefix-free?</td><td>Is Code B prefix-free?</td></tr>
<tr><td>Is Code A uniquely decodable?</td><td>Is Code B uniquely decodable?</td></tr>
<tr><td>Is Code A a Huffman code for some DMS?</td><td>Is Code B a Huffman code for some DMS?</td></tr>
<tr><td>The string 100010001110100 is encoded by Code A.
Decode it.</td><td>The string 01110001101 is encoded by Code B.
Decode it.</td></tr>
</table>

Problem 2. (8 pt) Consider a random variable X which has five possible values. Their probabilities are shown in the table below.

x	$p_X(x)$	Codeword $c(x)$	$\ell(x)$
a	0.30		
e	0.23		
c	0.20		
n	0.15		
t	0.12		

(a) (6 pt) Find a binary Huffman code (without extension) for this X. Put the values of the codewords and the codeword lengths in the table.

(b) (2 pt) Find the expected codeword length when Huffman coding is used without extension.

Problem 3. (7 pt) A memoryless source emits two possible messages Y(es) and N(o) with probabilities 0.2 and 0.8, respectively.

(a) (5 pt) Find the codeword lengths when Huffman binary code with second-order extension is used to encode this source. Put the values of the corresponding probabilities and the codeword lengths in the table below.

x_1x_2	$p_{X_1,X_2}(x_1,x_2)$	$\ell(x_1,x_2)$
YY		
YN		
NY		
NN		

(b) (2 pt) Find L_2. (This is the expected codeword length <u>per source symbol</u> of the Huffman binary code for the second-order extension of this source.)

Problem 4. (5.5 pt) [ENRPr] Consider a BSC whose crossover probability for each bit is $p = 0.45$. Suppose $P[X = 0] = 0.4$. Draw the channel diagram. Then, find the probability transition matrix $\mathbf{Q}$ and the joint pmf matrix $\mathbf{P}$.

Channel Diagram	$\mathbf{Q}$ matrix	$\mathbf{P}$ matrix

Problem 5. (13 pt) [ENRPr] Consider a DMC whose transition matrix $\mathbf{Q}$ is

$$\begin{array}{c c} x \setminus y & \begin{array}{cccc} 1 & \;\;2 & \;\;3 & \;\;4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.10 & 0.20 & 0.30 & 0.40 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.50 & 0.30 & 0.10 & 0.10 \end{bmatrix} \end{array}$$

Suppose the input probability vector is $\underline{\mathbf{p}} = [0.2, 0.4, 0.4]$.

In this problem, we will consider three decoders:

(a) The DIY decoder defined by $\hat{x}_{\text{DIY}}(y) = 2.5 - |y - 2.5|$.

(b) The MAP decoder.

(c) The ML decoder.

Find the decoding table for each of these decoders. Also, find their probabilities of decoding error. Put your answers in the table below.

y	$\hat{x}_{\text{DIY}}(y)$	$\hat{x}_{\text{MAP}}(y)$	$\hat{x}_{\text{ML}}(y)$
1			
2			
3			
4			
$P(\mathcal{E})$			

Problem 6. (10.5 pt) [ENRPr] For each of the following DMC's probability transition matrices $\mathbf{Q}$,

(i) (2 pt) indicate whether the corresponding DMC is weakly symmetric (Yes or No),

(ii) ($5.5 = 2 + 1 + 2 + 0.5^*$ pt.) evaluate the corresponding capacity value (your answer should be of the form X.XXXX), and

(iii)($3 = 1 + 1 + 0.5 + 0.5^*$ pt.) specify a channel input pmf (a row vector $\underline{\mathbf{p}}$) that achieves the capacity.

$\mathbf{Q}$	Weakly Symm.?	C	$\underline{\mathbf{p}}$
$\begin{bmatrix} 1/2 & 1/4 & 1/8 & 1/8 \\ 1/4 & 1/2 & 1/8 & 1/8 \\ 1/8 & 1/8 & 1/2 & 1/4 \\ 1/8 & 1/8 & 1/4 & 1/2 \end{bmatrix}$		_.____	
$\begin{bmatrix} 1/8 & 1/2 & 1/4 & 1/8 \\ 1/8 & 1/2 & 1/4 & 1/8 \end{bmatrix}$		_.____	
$\begin{bmatrix} 0 & 0.2 & 0.8 & 0 & 0 \\ 0.8 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$		_.____	
$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix}$		_.____	

Problem 7. (2 pt) [ENRPr] Consider a repetition code with a code rate of 1/9. Assume that a transmitter used this code to transmit an info-bit S over a BSC with crossover probability p. The receiver observes $\underline{\mathbf{y}} = 010101010$.

(a) Suppose $p = \frac{1}{\sqrt{3}}$. Then, $\hat{s}_{\mathrm{ML}}(010101010) =$ __.

(b) Suppose $p = \frac{1}{\sqrt{5}}$. Then, $\hat{s}_{\mathrm{ML}}(010101010) =$ __.

Problem 8. (8 pt) Consider a pair of random variables X and Y whose joint pmf matrix $\mathbf{P}$ is given by

$$\begin{array}{cc} x \setminus y & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1/8 & 0 & 0 & 1/8 & 0 \\ 1/8 & 1/8 & 1/8 & 0 & 1/8 \\ 0 & 1/8 & 0 & 1/8 & 0 \end{bmatrix} \end{array}$$

(a) (6 pt) [ENRPa] Evaluate the quantities in the table below.

$H(X,Y)$	
$H(X)$	
$H(Y)$	
$H(Y\|1)$	
$H(Y\|X)$	
$I(X;Y)$	

(b) (2 pt) Use the answer(s) in part (a) to determine whether X and Y are independent.

Problem 9. (1 pt)

(a) (1 pt) Do not forget to submit your study sheet with your exam.

(b) Reminder:

(i) Make sure that you write your name and ID on every page. (Read the instruction on the cover page.)

(ii) The online self-evaluation form is due by the end of today.